

# Kinetic model of GMSW in an anisotropic plasma

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## Abstract

A kinetic model of gravimagnetic shock waves (GMSW) in a locally anisotropic plasma is investigated. The equations of a drift approximation are written, and the moments of the distribution function are calculated. Solutions of the drift equations for a highly anisotropic ultrarelativistic plasma are found. It is shown that in this case the GMSW essentially affect the angular characteristics and the intensity of the magneto - bremsstrahlung of the magnetoactive plasma.

## 1 Introduction

In Ref. [1], from the requirement that the dynamical velocity of plasma be equal to that of the electromagnetic field due to the Einstein equations and the first group of the Maxwell equations, the equations of the relativistic magnetic hydrodynamics (RMHD) of magnetoactive plasma in a gravitational field were derived. In the same work (see also [2]) a remarkable class of exact solutions of the RMHD equations was obtained; it describes the motion of a magnetoactive locally isotropic plasma in the field of a plane gravitational wave (PGW) and is called gravimagnetic shock waves (GMSW). In Ref. [3] it was shown that in pulsar magnetospheres the GMSW are a highly effective detector of the gravitational radiation of neutron stars. An observed consequence of the energy transformation from the gravitational wave to the GMSW energy are the so-called giant pulses, sporadically arising in the radiation of a number of pulsars. The estimates made in [3] and [4] allow one to identify the giant pulses in the pulsar NP 0532 radiation with the gravitational radiation of this pulsar in the basic quadrupole mode of a neutron star.

A fundamental importance of GMSW for the gravitational theory leads to the necessity of a more detailed and comprehensive investigation of this phenomena. As was shown in [1] and [3], a GMSW is realized in an almost collisionless and nonequilibrium plasma situated in an abnormally strong magnetic field. Under these conditions, as a consequence of strong magneto - bremsstrahlung, the isotropy of the local distribution of plasma electrons is significantly violated, as was assumed in obtaining the solution in [1]. In [5], on the basis of the general RMHD equations, a hydrodynamic model of GMSW in an anisotropic plasma was built. Since in the hydrodynamical approach the number of equations obtained is smaller than that of unknown functions, we had to postulate a relation between the longitudinal and transverse components of the plasma pressure. In [5] the simplest variant of such a (linear) relation was studied. The study revealed a high dependence of the GMSW process on the degree of

plasma anisotropy, leading to the necessity of building a dynamical model of anisotropic magnetoactive plasma motion in the gravitaional radiation field. It is the problem dealt with in the present paper. Throughout the paper a set of units is used where  $(c = G = \hbar = 1)$ .

## 2 Drift solution to the kinetic equation for an anisotropic plasma

### 2.1 Field quantities in the plane gravitaional wave metric

Let us study a collisionless plasma in the PGW metric [6]:

$$ds^2 = 2dudv - L^2[e^{2\beta}(dx^2)^2 + e^{-2\beta}(dx^3)^2] \equiv 2dudv - A(dx^2)^2 - B(dx^3)^2, \quad (1)$$

where  $\beta(u)$  is an arbitrary function (the PGW amplitude), the function  $L(u)$  (the background factor of the PGW) obeys a second-order ordinary differential equation;  $u = \frac{1}{\sqrt{2}}(t - x^1)$  is the retarded time,  $v = \frac{1}{\sqrt{2}}(t + x^1)$  is the advanced time. The absolute future corresponds to the range  $T^+ : \{u > 0; v > 0\}$ , the absolute past to  $T^- : \{u < 0; v < 0\}$ . The metric (1) admits a group of motions  $\mathcal{G}_5$ , with the corresponding three linearly independent Killing vectors at a point:

$$\begin{aligned} \xi_{(1)}^i &= \delta_v^i; & \xi_{(2)}^i &= \delta_2^i; & \xi_{(3)}^i &= \delta_3^i. \end{aligned} \quad (2)$$

Let there be no GW for  $u \leq 0$ , i.e., –

$$\beta(u)|_{u \leq 0} = 0; \quad L(u)|_{u \leq 0} = 1, \quad (3)$$

and the homogeneous magnetic field be directed in the plane  $\{x^1, x^2\}$ :

$$\begin{aligned} H_{1|u \leq 0} &= H_0 \cos \Omega; & H_{2|u \leq 0} &= H_0 \sin \Omega; \\ H_{3|u \leq 0} &= 0; & E_{i|u \leq 0} &= 0, \end{aligned} \quad (4)$$

where  $\Omega$  is an angle between the  $0x^1$  axis (PGW propagation direction) and the magnetic field direction  $\mathbf{H}$ . The conditions (4) correspond to the vector potential

$$A_v = A_u = A_2 = 0; \quad A_3 = H_0(x^1 \sin \Omega - x^2 \cos \Omega); \quad (u \leq 0). \quad (5)$$

The electromagnetic field in the PGW metric (1) for an initially homogeneous plasma is described by the vector potential [1]:

$$\begin{aligned} A_2 &= A_v = A_u = 0; \\ A_3 &= -H_0 x^2 \cos \Omega + \frac{1}{\sqrt{2}} H_0 [v - \psi(u)] \sin \Omega, \end{aligned} \quad (6)$$

where  $\psi(u)$  is an arbitrary differentiable function satisfying the initial condition:

$$\psi|_{u \leq 0} = u. \quad (7)$$

In this case the only nonzero component of the Maxwell tensor depending on  $\psi$  is

$$F_{u3} = -\frac{1}{\sqrt{2}}H_0\psi' \sin \Omega. \quad (8)$$

Other nontrivial components of the Maxwell tensor are:

$$F_{23} = -H_0 \cos \Omega; \quad F_{v3} = \frac{1}{\sqrt{2}}H_0 \sin \Omega. \quad (9)$$

## 2.2 Collisionless kinetic equation

The collisionless kinetic equation for the 8-dimensional distribution function for charged particles of a kind  $a$ ,  $F_a(x^i, \mathcal{P}_i)$  has the form [7]:

$$[\mathcal{H}_a, F_a] \equiv \frac{\partial F_a}{\partial x^i} \frac{\partial \mathcal{H}_a}{\partial \mathcal{P}_i} + \frac{\partial F_a}{\partial \mathcal{P}_i} \frac{\partial \mathcal{H}_a}{\partial x^i} = 0, \quad (10)$$

where

$$\mathcal{P}_i = p_i + e_a A_i \quad (11)$$

is the generalized momentum of a particle and

$$\mathcal{H}_a(x^i, \mathcal{P}_i) = \frac{1}{2}g^{ij}(\mathcal{P}_i - e_a A_i)(\mathcal{P}_j - e_a A_j) \quad (12)$$

is the Hamiltonian function of a charged particle.

The process of obtaining equations in the drift approximation on the basis of the collisionless kinetic equations (10) was described by the author in [8]. Here we only somewhat transform the process for the case of an initially anisotropic distribution. Note, besides, that in the case  $\Omega \neq \pi/2$  the results [8] are erroneous because they use  $\mathcal{P}_2$  in the role of an integral of motion, which is the case only if  $\Omega = \pi/2$ . That was mentioned in the work cited above [1].

Let us study the case when the GW propagates perpendicularly to the magnetic field,  $\Omega = \pi/2$ , and let us look for solutions of the kinetic equation independent of the variables  $v, x^2$  and  $x^3$ . As in [8], let us introduce the unit timelike vector  $v^i$ :

$$v_2 = v_3 = 0; \quad v_u = \sqrt{\frac{\psi'}{2}}; \quad v_v = \sqrt{\frac{1}{2\psi'}} \quad (13)$$

and transform the kinetic equation into the frame of reference (FR) moving with the velocity  $v^i$ :

$$\mathcal{P}_v = v_u(\mathcal{P}_4 + \mathcal{P}_1) \quad \mathcal{P}_u = v_v(\mathcal{P}_4 - \mathcal{P}_1). \quad (14)$$

The Jacobian of this transformation is equal to unity:

$$\frac{D(\mathcal{P}_v, \mathcal{P}_u)}{D(\mathcal{P}_1, \mathcal{P}_4)} = 1. \quad (15)$$

As a result, we get the equation:

$$\begin{aligned} v_v(\mathcal{P}_4 + \mathcal{P}_1) \frac{\partial F_a}{\partial u} - v'_v(\mathcal{P}_4 + \mathcal{P}_1) \left( \mathcal{P}_1 \frac{\partial F_a}{\partial \mathcal{P}_4} + \mathcal{P}_4 \frac{\partial F_a}{\partial \mathcal{P}_1} \right) - \frac{eH}{\sqrt{B}} \mathcal{P}_3 \frac{\partial F_a}{\partial \mathcal{P}_1} \\ + \frac{1}{2} [(A^{-1})' \mathcal{P}_2^2 + (B^{-1})' \mathcal{P}_3^2] v_u \left( \frac{\partial F_a}{\partial \mathcal{P}_4} + \frac{\partial F_a}{\partial \mathcal{P}_1} \right) = 0, \end{aligned} \quad (16)$$

where:

$$H^2 = -\frac{2}{B} \partial_u A_3 \partial_v A_3 = H_0^2 \psi' \frac{e^{2\beta}}{L^2} \quad (17)$$

is an invariant of the electromagnetic field (the magnetic field intensity squared in the FR moving with the velocity  $v^i$ ).

On the PGW front the solution (16) must satisfy the initial condition corresponding to a homogeneous anisotropic currentless plasma:

$$F_a(x^i, \mathcal{P}_i)|_{u=0} = f_a(p_\perp^2, p_\parallel^2) \delta(\mathcal{H}_a - \frac{1}{2} m_a^2), \quad (18)$$

where  $f_a$  is an arbitrary function of its arguments and the following notations are introduced:

$$p_\perp^2 = p_1^2 + p_3^2; \quad p_\parallel = -p_2. \quad (19)$$

The kinetic equation (16) has three exact integrals:

$$\begin{aligned} \mathcal{H}_a(x^i, \mathcal{P}_i) &= \frac{1}{2} m_a^2 = \text{Const}; \\ \mathcal{P}_2 &= \text{Const}; \quad \mathcal{P}_3 = \text{Const}. \end{aligned} \quad (20)$$

For a complete solution of the problem we need one more independent integral.

### 2.3 Drift approximation

We shall solve Eq. (16) in the drift approximation, when the Larmor frequency for each kind of charged particles –

$$\omega_a = \frac{e_a H}{m_a} \quad (21)$$

is much greater than the characteristic frequency  $\omega$  of the gravitational wave:

$$\Lambda = \frac{\omega}{\omega_a} \ll 1. \quad (22)$$

In the zeroth order with respect to the parameter  $\Lambda$  (16) takes the form:

$$H \frac{\partial F_a}{\partial \mathcal{P}_1} = 0, \quad (23)$$

i.e. in the drift approximation  $F_a$  is independent obviously of  $\mathcal{P}_1$ . Thus in the drift approximation, apart from the above exact integrals, the kinetic equation has also drift (approximate) integrals [8]:

$$\mathcal{P}_4 \approx \text{Const}; \quad u \approx \text{Const}. \quad (24)$$

The solution of the kinetic equation corresponding to the drift approximation, which, in the absence of GW, is transformed to an anisotropic and currentless one, can be written in the form

$$F_a = f_a (\mathcal{P}_4^2 - \mathcal{P}_2^2, \mathcal{P}_2^2, u) \delta(\mathcal{H}_a - \frac{1}{2}m_a^2), \quad (25)$$

where  $f_a$  is an arbitrary function of its arguments. In particular, the following  $f_a$  can be chosen:

$$f_a = f_a^0 \left[ \mu_\perp^{-2} (\mathcal{P}_4^2 - \mathcal{P}_2^2 - m_a^2) + \mu_\parallel^{-2} \mathcal{P}_2^2 \right], \quad (26)$$

where  $\mu_\parallel(u)$  and  $\mu_\perp(u)$  are arbitrary functions of their arguments. Thus in the FR (14) moving with the velocity  $v_i$  the drift solution locally coincides with the unperturbed distribution (18).

Substituting the obtained distribution function of the zero drift approximation (25) into the kinetic equation (16), it is easy to derive a correction of the first drift approximation,  $\delta F_a \sim \Lambda^{-1}$ , for the distribution function determined up to an additive component being an arbitrary function of the above drift integrals (see [8]). However, exact consequences of the collisionless kinetic equation (16) are conservation laws for the number of each kind of particles and the total energy-momentum tensor (EMT) of the plasma and the electromagnetic field [9]. These laws impose certain restrictions on the above additive component and lead to differential equations for the functions  $\mu_\parallel(u)$  and  $\mu_\perp(u)$ .

Due to the symmetry properties, it turns out that the first order correction to the distribution function contributes only to the component  $n_3$  of the particle number current density vector and the components  $T_{v3}^a$  and  $T_{u3}^a$  of the particle EMT. However, the component  $n_3(u, v)$  does not affect the continuity equation, only the above two components of the particle EMT appear in the transport equation of the total EMT for the component  $i = 3$  they do not appear in other transport equations. On the other hand, the drift current determined by the first correction to the distribution function can be obtained as a consequence of the energy-momentum conservation laws and the Maxwell equations, without addressing to a solution of the kinetic equation (see [1]). As a result, it turns out that the set of the transport equations and the Maxwell equations split into two subsets, one of which, determined by the zero drift approximation, is self-consistent and closed and entirely determines the functions  $\psi(u)$ ,  $\mu_\parallel(u)$  and  $\mu_\perp(u)$ . The correction of the first drift approximation to the distribution function does not affect the magnetic hydrodynamics equation.

### 3 Derivation of the magnetic hydrodynamics equations

#### 3.1 Algebraic structure of the distribution function moments

Returning to the original FR by means of (14), let us introduce the following scalars:

$$\begin{aligned} p_{\parallel} &= (p, n); & p^2 &= (v^i v^j - g^{ij}) p_i p_j; \\ p_{\perp} &= p^2 - p_{\parallel}^2 = (v^i v^j - g^{ij} - n^i n^j) p_i p_j, \end{aligned} \quad (27)$$

where  $n^i$  is the unit spacelike vector in the direction of the magnetic field intensity vector  $H_i = v^j \overset{*}{F}_{ji}$ :

$$n_i = \frac{H_i}{H}; \quad H^2 = -(H, H). \quad (28)$$

then the distribution function in the zero drift approximation (26) takes the form

$$f_a = f_a^0 (\mu_{\parallel}^{-2} p_{\parallel}^2 + \mu_{\perp}^{-2} p_{\perp}^2). \quad (29)$$

It is not difficult to show that the moments of this distribution are:

$$\begin{aligned} n_a^i(x) &= \int_{P(x)} f_a(x, p) p^i dP; \\ T_a^{ij}(x) &= \int_{P(x)} f_a(x, p) p^i p^j dP, \end{aligned} \quad (30)$$

where

$$dP = \frac{\sqrt{-g}(2S+1)dp^1 dp^2 dp^3}{(2\pi\hbar)^3 p_4}$$

( $S$  - is the particle spin), and have the following algebraic structure:

$$n_a^i(u) = n v^i; \quad (31)$$

$$T_a^{ij}(u) = (\varepsilon + P_{\perp}) v^i v^j - P_{\perp} g^{ij} + (P_{\parallel} - P_{\perp}) n^i n^j, \quad (32)$$

where  $n$ ,  $\varepsilon$ ,  $P_{\parallel}$  and  $P_{\perp}$  are some scalar functions of the variable  $u$ ; the indices of the kind of particles,  $a$ , are dropped in these scalars for the sake of simplicity of notations. The EMT track of particles is equal to:

$$T_a(u) = \varepsilon - 2P_{\perp} - P_{\parallel}. \quad (33)$$

From (31) it follows that the velocity vector  $v_i$  introduced in (13) coincides with the plasma kinematic velocity vector. It is not difficult to check the fulfilment of the condition [1]:

$$(v, n) = 0, \quad (34)$$

and consequently the vector  $v^i$  is an eigenvector of the particle EMT, i.e. it is the dynamic plasma velocity vector according to Synge [10]. thus the frame of reference introduced by the relations (14) is comoving the plasma.

Note that in the first drift approximation the equation for kinematic and dynamic velocities of particles is violated and therefore a drift current arises.

### 3.2 Magnetic hydrodynamics equation for an anisotropic plasma in the PGW field

In Sec. 3 of Ref. [1] the RMHD equations for a plasma in an arbitrary gravitational field and for an arbitrary structure of the plasma EMT,  $\overset{p}{T}_{ij}$ , with the eigenvector  $v^i$  were obtained:

$$\overset{p}{T}_{ij} v^j = \varepsilon v_i, \quad (35)$$

where the invariant  $\varepsilon > 0$  is the plasma energy density in the comoving FR. In this case the following conditions were imposed on the invariants of the electromagnetic field:

$$F^{ij} \overset{*}{F}_{ij} = 0; \quad (36)$$

$$F_{ij} F^{ij} = 2H^2 > 0. \quad (37)$$

In [1] it was shown that, under these conditions, from the conservation law

$$T^{ij}_{,j} = 0 \quad (38)$$

for the whole EMT of the plasma and the electromagnetic field

$$T_{ij} = \overset{p}{T}_{ij} + \overset{f}{T}_{ij}$$

and the first group of the Maxwell equations

$$\overset{*}{F}{}^{ij}_{,j} = 0 \quad (39)$$

it follows:

1. The conditions of embedding of the magnetic field in the plasma are:

$$F_{ij} v^j = 0 \quad (40)$$

(in this case the velocity vector  $v_i$  also automatically becomes an eigenvector of the electromagnetic field EMT);

2. The second group of the Maxwell equations is

$$F_{,j}^{ij} = -4\pi J_{dr}^i \quad (41)$$

with the drift current

$$J_{dr}^i = -\frac{2F^{ik} T_{k,l}^l}{F_{jm} F^{jm}}; \quad (42)$$

3. The differential relations are

$$v^i T_{i,k}^k = 0, \quad (43)$$

$$H^i T_{i,k}^k = 0. \quad (44)$$

It is not difficult to ascertain that the Maxwell tensor determined by the vector potential (6) automatically satisfies the conditions (37) and the velocity vector 13) – the embedding conditions (40). Therefore in this case, due to the EMT conservation (39), which is an exact consequence of the kinetic equations, the relations (41) — (44) must hold. The isotropic Killing vector (2) gives an exact integral of Eqs. (39): (39):

$$L^2 T_v^u = \text{Const}. \quad (45)$$

thus, taking into account Eqs. (13) and (32) and the initial conditions (3) and (7), we get from (45):

$$L^2(\varepsilon + P_\perp) = \psi'(\varepsilon + P_\perp) \Delta(u). \quad (46)$$

where we have introduced the so-called *GMSW governing function* (see [1]):

$$\Delta(u) = 1 - \alpha^2(e^{2\beta} - 1) \quad (47)$$

and the *GMSW dimensionless parameter*

$$\alpha^2 = \frac{H_0^2}{4\pi(\varepsilon + P_\perp)}. \quad (48)$$

It can be further shown that the relation (44) is transformed into an identity, and Eq. (43) gives:

$$\varepsilon' + (\ln H)'(P_\parallel - P_\perp) - \frac{1}{2} \left( \ln \frac{\psi'}{L^4} \right)' (\varepsilon + P_\parallel) = 0. \quad (49)$$

The EMT conservation equation and the Maxwell equations do not give other independent relations.



The missing equations are obtained from the particle number conservation laws which are also an exact consequence of the kinetic equations:

$$\begin{aligned} n_{,i}^i &= L^{-2} (L^2 n v_v)' = 0 \Rightarrow \\ n_a(u) L^2 &= \sqrt{\psi'} n_a^0. \end{aligned} \quad (50)$$

Thus we get the set of equations (46), (49) and (50) for determining the three unknown scalar functions  $\psi$ ,  $\mu_{\parallel}$  and  $\mu_{\perp}$ . It is only necessary to obtain in this way explicitly the scalars  $n$ ,  $\varepsilon$ ,  $P_{\parallel}$  and  $P_{\perp}$  out of  $(1 + 2n)$  scalars, where  $n$  is the particle kind number.

Note that there are at least two kinds of charged particles (in the case of interest these are protons and electrons) in the plasma. Thus, there are two kinds of scalars  $\mu_{\parallel}$  and  $\mu_{\perp}$ , and one particle number conservation law for a given kind of particles connects each couple of them (50). The summed components of pressure and energy density appear in equations (46) and (49). As a consequence of the initial electroneutrality of the plasma, (50), we get a relation between the local concentrations of neutrons and protons [8]:

$$n_e(u) = n_p(u). \quad (51)$$

### 3.3 Calculating the moments of the distribution function

For calculating the above scalar functions it is necessary to find the moments of the distribution function (30). An easiest way to do it is to use Eqs. (31) and (32) in the comoving FR according to (14), using the property (15) of this transformation and using the spherical coordinates in the momentum space:

$$\begin{aligned} p_1 &= \mu_{\perp} p \cos \theta \cos \phi; \\ p_3 &= \mu_{\perp} p \cos \theta \sin \phi; \\ p_2 &= \mu_{\parallel} p \sin \theta. \end{aligned} \quad (52)$$

In so doing we obtain:

$$n = n_0 \mu_{\perp}^2 \mu_{\parallel}, \quad (53)$$

where

$$n_0 = \frac{2S+1}{2\pi^2} \int_0^{\infty} f(p^2) p^2 dp, \quad (54)$$

the integration variable is:

$$p^2 = \mu_{\parallel}^{-2} p_{\parallel}^2 + \mu_{\perp}^{-2} p_{\perp}^2. \quad (55)$$

thus we obtain from (53), (50) and (51):

$$(\mu_{\parallel}^2 \mu_{\perp})_e = (\mu_{\parallel}^2 \mu_{\perp})_p = \frac{\sqrt{\psi'}}{L^2} (\mu_{\parallel}^2 \mu_{\perp})_0. \quad (56)$$

Further, setting for definiteness

$$\mu_{\perp} \leq \mu_{\parallel},$$

we get the expressions for the components of the plasma pressure and energy density:

$$P_{\parallel} = \frac{(2S+1)\mu_{\perp}^2\mu_{\parallel}^3}{4\pi^2(\mu_{\parallel}^2 - \mu_{\perp}^2)} \int_0^{\infty} f(p^2) p^2 dp \times \left[ \sqrt{m^2 + \mu_{\parallel}^2 p^2} - \frac{m^2 + \mu_{\perp}^2 p^2}{p\sqrt{\mu_{\parallel}^2 - \mu_{\perp}^2}} \ln \left( \frac{\sqrt{m^2 + \mu_{\parallel}^2 p^2} + p\sqrt{\mu_{\parallel}^2 - \mu_{\perp}^2}}{\sqrt{m^2 + \mu_{\perp}^2 p^2}} \right) \right]; \quad (57)$$

$$P_{\perp} = \frac{(2S+1)\mu_{\perp}^4\mu_{\parallel}}{8\pi^2(\mu_{\parallel}^2 - \mu_{\perp}^2)} \int_0^{\infty} f(p^2) p^2 dp \times \left[ -\sqrt{m^2 + \mu_{\parallel}^2 p^2} + \frac{m^2 + p^2(2\mu_{\parallel}^2 - \mu_{\perp}^2)}{p\sqrt{\mu_{\parallel}^2 - \mu_{\perp}^2}} \ln \left( \frac{\sqrt{m^2 + \mu_{\parallel}^2 p^2} + p\sqrt{\mu_{\parallel}^2 - \mu_{\perp}^2}}{\sqrt{m^2 + \mu_{\perp}^2 p^2}} \right) \right]; \quad (58)$$

$$\varepsilon = \frac{2S+1}{4\pi^2} \mu_{\perp}^2 \mu_{\parallel} \int_0^{\infty} f(p^2) p^2 dp \times \left[ \sqrt{m^2 + \mu_{\parallel}^2 p^2} + \frac{m^2 + \mu_{\perp}^2 p^2}{p\sqrt{\mu_{\parallel}^2 - \mu_{\perp}^2}} \ln \left( \frac{\sqrt{m^2 + \mu_{\parallel}^2 p^2} + p\sqrt{\mu_{\parallel}^2 - \mu_{\perp}^2}}{\sqrt{m^2 + \mu_{\perp}^2 p^2}} \right) \right]. \quad (59)$$

Note that all the expressions (57) – (59) have finite limits at  $\mu_{\parallel}^2 - \mu_{\perp}^2 = 0$ . From (57) – (59) one can obtain:

$$\varepsilon - P_{\parallel} - 2P_{\perp} = \frac{(2S+1)m^2\mu_{\perp}^2\mu_{\parallel}}{2\pi^2\sqrt{\mu_{\parallel}^2 - \mu_{\perp}^2}} \int_0^{\infty} \ln \left( \frac{\sqrt{m^2 + \mu_{\parallel}^2 p^2} + p\sqrt{\mu_{\parallel}^2 - \mu_{\perp}^2}}{\sqrt{m^2 + \mu_{\perp}^2 p^2}} \right) f(p^2) p dp. \quad (60)$$

Besides, note that in the case  $\mu_{\parallel} \leq \mu_{\perp}$  it is essential that the logarithmic functions in the above formulae for the moments are to be substituted by the functions arcsin.

## 4 Ultrarelativistic plasma

### 4.1 Energy density and pressure

magnetosphere (see, e.g., [11]), leads to very high values of the kinetic energy of electrons and protons (up to  $10^{18}$  eV). Thus GMSW appear to be always realized in an ultrarelativistic plasma.

In the ultrarelativistic plasma the particle rest mass does not affect the macroscopic moments in the drift approximation, therefore we can set:

$$(\mu_{\parallel})_e = (\mu_{\parallel})_p; \quad (\mu_{\perp})_e = (\mu_{\perp})_p. \quad (61)$$

For an ultrarelativistic plasma,

$$\mu_{\perp} p \gg m; \quad \mu_{\parallel} p \gg m; \quad (62)$$

the above expressions become explicit functions of  $\mu_{\perp}$  and  $\mu_{\parallel}$ :

$$P_{\parallel} = \frac{E_0}{2} \frac{\mu_{\perp}^2 \mu_{\parallel}^3}{\mu_{\parallel}^2 - \mu_{\perp}^2} \left[ \mu_{\parallel} - \frac{\mu_{\perp}^2}{\sqrt{\mu_{\parallel}^2 - \mu_{\perp}^2}} \ln \left( \frac{\mu_{\parallel} + \sqrt{\mu_{\parallel}^2 - \mu_{\perp}^2}}{\mu_{\perp}} \right) \right]; \quad (63)$$

$$P_{\perp} = \frac{E_0}{4} \frac{\mu_{\perp}^4 \mu_{\parallel}}{\mu_{\parallel}^2 - \mu_{\perp}^2} \left[ \frac{2\mu_{\parallel}^2 - \mu_{\perp}^2}{\sqrt{\mu_{\parallel}^2 - \mu_{\perp}^2}} \ln \left( \frac{\mu_{\parallel} + \sqrt{\mu_{\parallel}^2 - \mu_{\perp}^2}}{\mu_{\perp}} \right) - \mu_{\parallel} \right], \quad (64)$$

$$\varepsilon = \frac{E_0}{2} \mu_{\perp}^2 \mu_{\parallel} \left[ \mu_{\parallel} + \frac{\mu_{\perp}^2}{\sqrt{\mu_{\parallel}^2 - \mu_{\perp}^2}} \ln \left( \frac{\mu_{\parallel} + \sqrt{\mu_{\parallel}^2 - \mu_{\perp}^2}}{\mu_{\perp}} \right) \right], \quad (65)$$

where

$$E_0 = \sum_a \frac{2S+1}{2\pi^2} \int_0^{\infty} f(p^2) p^3 dp = \text{Const} \quad (66)$$

is the energy density of an isotropic homogeneous plasma. From (60), in the approximation under study it follows:

$$\varepsilon - P_{\parallel} - 2P_{\perp} = 0. \quad (67)$$

## 4.2 Special solutions

With the expressions obtained, it is still very difficult to extract any information from Eqs. (46) and (49). Therefore let us study these equations at the limiting values of the plasma anisotropy parameter:

$$\delta = \frac{\mu_{\perp}}{\mu_{\parallel}}. \quad (68)$$

### 4.2.1 Isotropic plasma: $\delta \rightarrow 1$

Calculating the limits of the expressions (62) - (66) as  $\delta \rightarrow 1$ , we get:

$$P_{\parallel} = P_{\perp} = \frac{1}{3} \varepsilon = \frac{1}{6} \mu^4 E_0, \quad (69)$$

where  $\mu_{\parallel} = \mu_{\perp} = \mu$ . Then it is easy to verify that Eqs. (46) and (56) have similar solutions:

$$\mu^3 = \frac{\mu_0^3}{L^2} \sqrt{\psi'}. \quad (70)$$

This fact is of fundamental importance as it provides a microscopic explanation to RMHD [1]. In reality, if Eqs. (46) and (56) had independent consequences, it would indicate that the initial anisotropy of a collisionless plasma is broken by a gravitational wave, i.e. the RMHD equations would have been ineligible for the description of the collisionless plasma.

Substituting relation (70) into Eq. (49), we get the known solutions [1] for an isotropic ultrarelativistic plasma:

$$\psi' = \frac{1}{L^2 \Delta^3}; \quad (71)$$

$$\varepsilon = \frac{\varepsilon_0}{L^4 \Delta^2}; \quad (72)$$

$$H^2 = H_0^2 \frac{e^{2\beta}}{L^4 \Delta^3}; \quad (73)$$

in this case

$$\mu = \frac{\mu_0}{L} \frac{1}{\sqrt{\Delta}}. \quad (74)$$

#### 4.2.2 Plasma with a chilled transverse momentum: $\delta \rightarrow 0$

In this case from (62) - (66) we obtain:

$$P_{\parallel} \approx \varepsilon = \frac{1}{2} E_0 \mu_{\parallel}^2 \mu_{\perp}^2; \quad P_{\perp} \approx 0. \quad (75)$$

Then Eq. (46) has an integral:

$$\frac{\varepsilon H L^4}{\psi'} = \varepsilon_0 H_0. \quad (76)$$

Using in (76) the integral (56) and the expression for  $H$  (17), we obtain an expression for  $\mu_{\perp}$ :

$$\mu_{\perp} = (\mu_{\perp})_0 \frac{e^{-\beta}}{L}. \quad (77)$$

Now, substituting (77) into (75) and the result obtained into (46), using the integral (56), we get:

$$\begin{aligned} \psi' &= \frac{e^{-2\beta}}{L^2 \Delta^2}; \\ \mu_{\parallel} &= \frac{(\mu_{\parallel})_0}{L \sqrt{\Delta}}; \end{aligned} \quad (78)$$

$$\varepsilon = P_{\parallel} = \varepsilon_0 \frac{e^{-2\beta}}{L^4 \Delta}; \quad P_{\perp} = \overset{0}{P}_{\perp} \frac{e^{-4\beta}}{L^4}; \quad (79)$$

$$H^2 = \frac{H_0^2}{L^4 \Delta^2}. \quad (80)$$

From the solutions presented it is obvious that on the GMSW front ( $\Delta \rightarrow 0$ ) the transverse pressure component is almost conserved ( $P_{\perp} \approx \overset{0}{P}_{\perp}$ ), whereas the longitudinal pressure component tends to infinity proportionally to  $\Delta^{-1}$ . thus, on the GMSW front the plasma initial anisotropy (in case it exists) intensifies.

## 5 Anisotropy effect on the magneto - bremsstrahlung

The squared projection values, averaged over the distribution (29) in the co-moving FR, are of the order

$$\langle p_{\perp}^2 \rangle \sim \mu_{\perp}^2; \quad \langle p_{\parallel}^2 \rangle \sim \mu_{\parallel}^2. \quad (81)$$

therefore the average value of the ultrarelativistic particle energy is

$$\langle \mathcal{E} \rangle = \sqrt{p_{\perp}^2 + p_{\parallel}^2} \sim \sqrt{\mu_{\perp}^2 + \mu_{\parallel}^2}. \quad (82)$$

thus in the case of a highly anisotropic plasma (for  $\delta \rightarrow 0$ ) we obtain from (82):

$$\langle \mathcal{E} \rangle \sim \mu_{\parallel}. \quad (83)$$

Substituting  $\mu_{\perp}$  and  $\mu_{\parallel}$  from (77) and (78) into these expressions, we get:

$$\begin{aligned} \langle p_{\perp}^2 \rangle &\approx \langle p_{\perp}^2 \rangle_0 \frac{e^{-2\beta}}{L^2} \approx \text{Const}; \\ \langle p_{\parallel}^2 \rangle &\approx \frac{\langle p_{\parallel}^2 \rangle_0}{L^2 \Delta}; \\ \langle \mathcal{E} \rangle &\approx \frac{\langle \mathcal{E} \rangle_0}{L \sqrt{\Delta}}. \end{aligned} \quad (84)$$

At small values of  $\delta$  the average value of the angle  $\chi$  between the vectors of microscopic velocity of an ultrarelativistic particle  $\mathbf{V}$  and the magnetic field intensity  $\mathbf{H}$  is small as well:

$$\sin \chi = \frac{V_{\perp}}{V} \approx \frac{\sqrt{\langle p_{\perp}^2 \rangle}}{\langle \mathcal{E} \rangle} \Rightarrow \chi \approx \chi_0 e^{-\beta} \sqrt{\Delta}, \quad (85)$$

where  $\chi_0 = \delta$ .

As is known (see e.g. [12]), the magnetobremsstrahlung intensity for an ultrarelativistic particle is concentrated in a narrow cone with the axis  $\mathbf{H}$  and the angle  $2\chi$  at the top. Thus, if  $2\chi_0$  is the output angle of magnetobremsstrahlung

of a nonperturbed plasma, then  $2\chi$  is the output angle of magnetobremssstrahlung in the GMSW. On the GMSW front ( $\Delta(u) \rightarrow 0$ ) the output angle of the radiation tends to zero. In this case the total intensity of the magnetobremssstrahlung of an electron[12]:

$$I = \frac{2e^4}{3m^4} p_{\perp} H^2 \quad (86)$$

according to (80 and (84) in the GMSW is equal to:

$$I(u) = I_0 \frac{e^{-2\beta(u)}}{L^6(u) \Delta^2(u)} \quad (87)$$

(where  $I_0$  is an unperturbed radiation intensity) and tends to infinity on the GMSW front.

If the output angle of the magnetobremssstrahlung of the pulsar magnetosphere is connected with the duration of the radiation pulse, then, as the gravitational wave travels through the pulsar magnetosphere, a strong contraction of the pulse with a simultaneous increase of its intensity, will be observed. This effect offers an Alternative, as compared to [3] and [4], opportunity to explain the giant pulses of the pulsars, whose radiation direction diagram is in most cases pencil-like.

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